Packing tight Hamilton cycles

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Carnegie Mellon University

Joint work with Alan Frieze and Michael Krivelevich

General graphs

Definition

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Remark

Cannot hope to pack regular graphs with H-cycles if $d < \frac{n}{2}$.

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Erdős-Rényi $G_{n,p}$: edges appear independently with probability p.

- (Komlós, Szemerédi '83; Koršunov '77.) $G_{n,p}$ is Hamiltonian **whp** if $p = \frac{\log n + \log\log n + \omega(n)}{n}$ with $\omega(n) \to \infty$.

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- (Bollobás, Frieze '85.) For any fixed k, **whp** k disjoint H-cycles appear as soon as all degrees are at least $2k$.
- (Kim, Wormald '01.) For fixed r, random r-regular graphs contain $\lfloor \frac{r}{2} \rfloor$ disjoint H-cycles **whp**.

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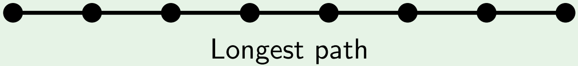
Pósa rotation



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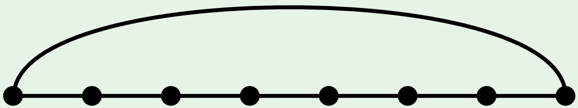

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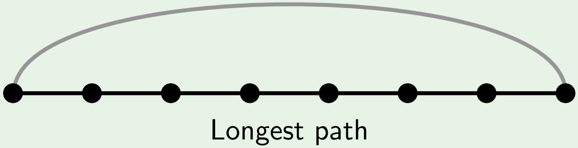

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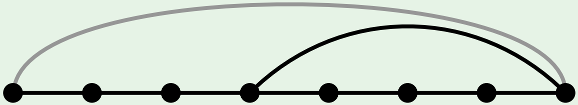

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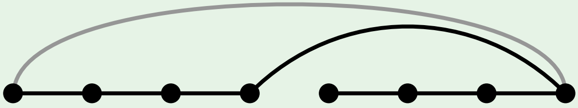

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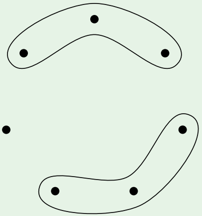
Definition (3-uniform hypergraph)

$H_{n,p;3}$: each triple appears independently with probability p.



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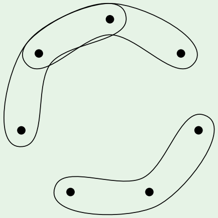

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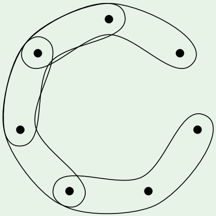

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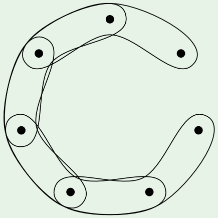

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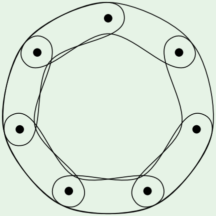

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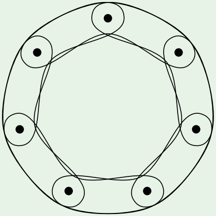

Tight H-cycle

Loose H-cycle

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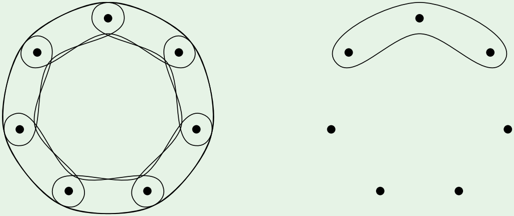

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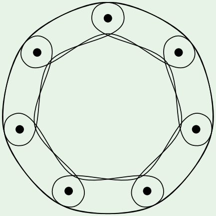

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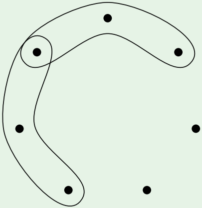

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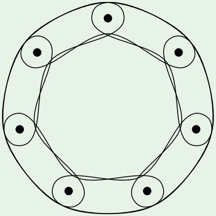

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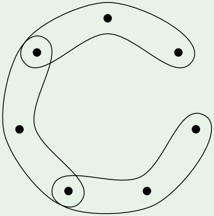

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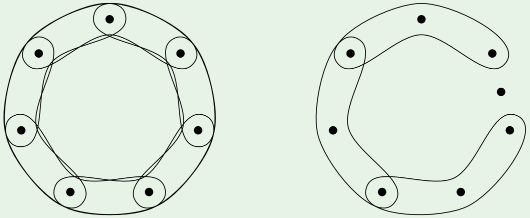

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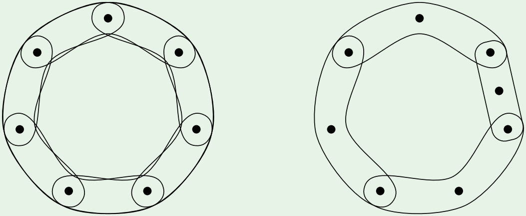

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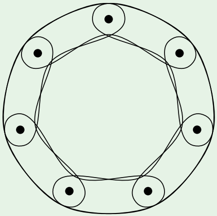

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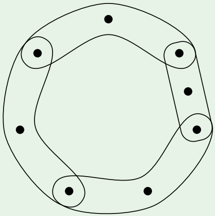

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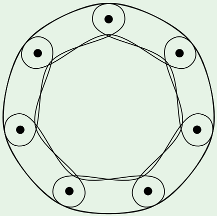

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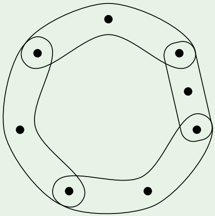

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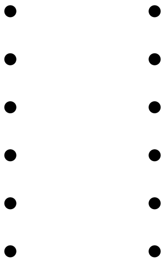

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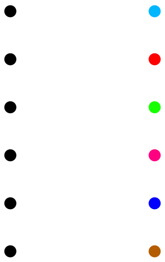

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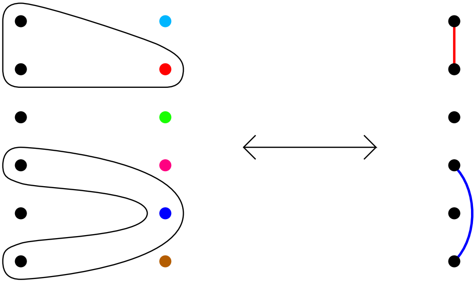

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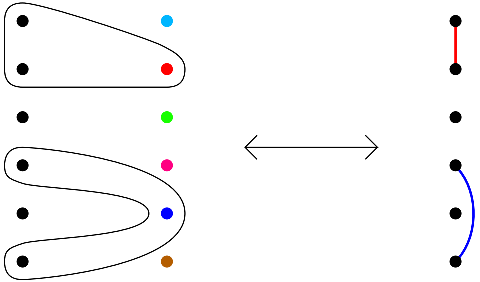

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Living without the Pósa lemma

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THEOREM (COOPER, FRIEZE '02)

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Theorem (Frieze, L. '10)

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Packing H-cycles in hypergraphs

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- *Pseudorandom* 3-uniform hypergraphs with $4 \mid n$ can be almost packed with *tight* H-cycles.
- *Pseudorandom* directed graphs with $2 \mid n$ can be almost packed with H-cycles.

THEOREM (CHUNG, GRAHAM, WILSON '89)

For fixed p, equivalent properties:

- Every set U spans $\frac{p}{2}|U|^2 + o(n^2)$ edges.

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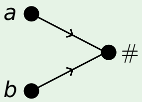

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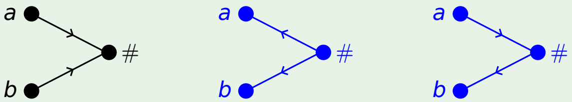

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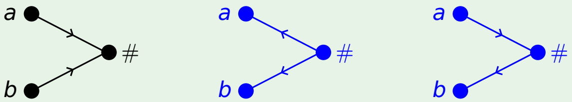

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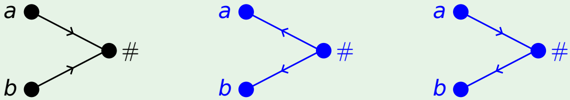

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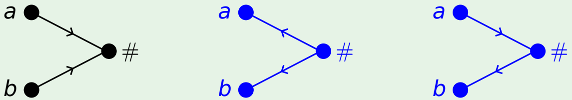

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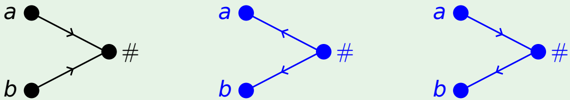

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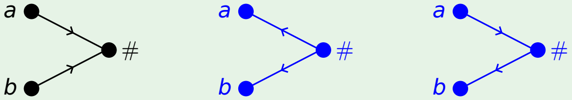

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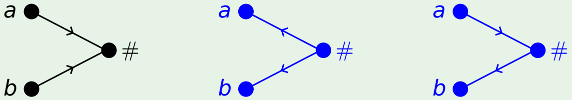

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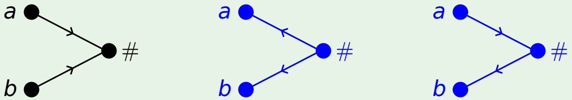

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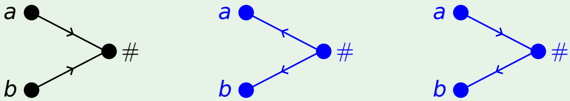

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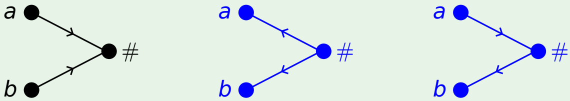

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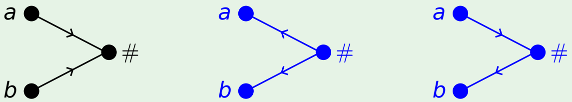

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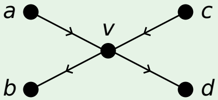
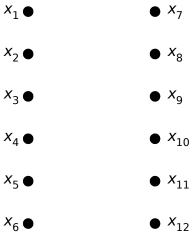


Reduction: digraphs to bipartite graphs

Link H-cycles in digraphs to perfect matchings in bipartite graphs:

- Randomly split and permute the digraph vertices.



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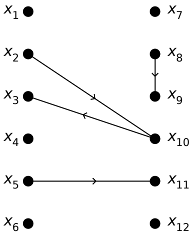

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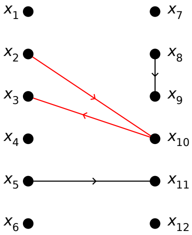

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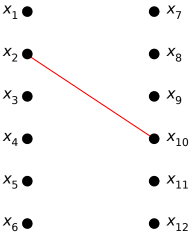

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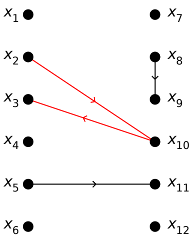

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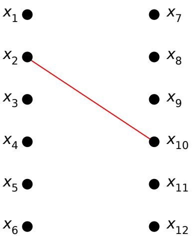


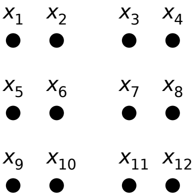
Auxiliary bipartite graph

- Show the auxiliary graph is also pseudorandom **whp** (all degrees and co-degrees are correct).
- Pack perfect matchings in the auxiliary graph; recover as Hamilton cycles in the digraph.

Reduction: hypergraphs to digraphs

Link tight H-cycles in 3-graphs to H-cycles in digraphs:

- Randomly permute vtxs, and make consecutive *ordered* pairs.



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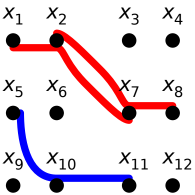

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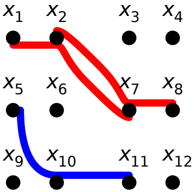

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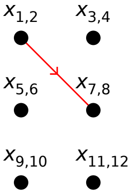

Auxiliary digraph

- Place $\overrightarrow{x_{1,2}x_{7,8}}$ if both $\{x_1, x_2, x_7\}, \{x_2, x_7, x_8\}$ are hyperedges.

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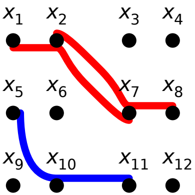

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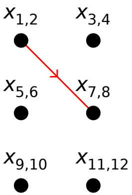

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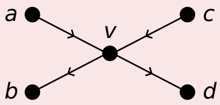
Questions

- Is the (a, b, c, d) condition necessary for pseudorandom digraph packing?



- Remove divisibility conditions from results.
- Generalize to higher uniformity.